GCSE Maths – Probability

Enumeration, Venn Diagrams, Tree Diagrams and Tables

Notes

WORKSHEET

Enumeration

An enumeration is an **ordered** and **complete** list of items in a population. In GCSE maths, this means knowing how to list combinations and sets of events, as well as using theoretical probability to find the likelihood of specified events.

Systematic Listing

Possible combinations can be found by listing. If you have the numbers 1, 2 and 3 and want to find out how many 3-digit numbers can be made, start by listing all the three-digit numbers beginning with 1, then 2, then 3: 123, 132, 213, 231, 312, 321. By systematically working through the options, we can be sure that no combinations have been missed.

Product Rule (Higher Only)

When finding the number of **possible combinations** or combined outcomes, the product rule can be applied. The **product rule** is:

If there are x outcomes of Event 1 and y outcomes of Event 2, then there xy combined outcomes.

In a mathematical problem, this means **multiplying all the possibilities** of multiple events together.

For example, if you have 10 shirts and 8 pairs of trousers, then you could wear $\mathbf{80}$ (10×8) **different** outfits. This applies for any number of events, as long as **none of them are limited** – if you have already chosen a shirt, then your possible choices of shirt **reduce** to 1. There are now only $\mathbf{8}$ (1×8) possible outfits you could wear.

Similarly, if you have 10 shirts, 8 pairs of trousers and 4 ties, then you could wear $\mathbf{240}$ $(10 \times 8 \times 4)$ different outfits.

Systematic listing can be **represented** in different ways, including lists, tables and diagrams.

Probability Notation

These are some key terms and ideas in probability – many of these are covered in more detail in the other Probability revision notes.

- Theoretical probability is the mathematical **likelihood** of something happening. It is always given a **value between 0 and 1**.
- An event is the **outcome** of an action or experiment.
- The **sample space** is all possible outcomes.
- The **target outcome** is the outcome that we want to find the probability of occurring.
- The **possible outcomes** are all the outcomes in the sample space.
- A **population** is the overall group of people, values or items that can be **grouped by a common characteristic**.
- A **sample** is a smaller sub-group of a population.

Types of Events

Events are **independent** if the probability of one outcome occurring **does not** affect the probability of another outcome occurring.

Events are **dependent** if the probability of one outcome occurring **does** affect the probability of another outcome occurring.

Events are **mutually exclusive** if they cannot **both** occur.

Probability Notation

As well as key terms, we can use **notation** to describe events and their theoretical probability.

- $P(A)$ means the probability of outcome A.
- $P(A \cup B)$ means the probability of outcome A **or** outcome B.
- $P(A \cap B)$ means the probability of outcome A **and** outcome B.
- $P(B')$ means the probability of **not** outcome B.

Set Notation

Set notation is used in mathematics to show a **list of outcomes**.

Set notation uses curly brackets {} known as braces. Everything listed inside the braces is an **element** of the set.

- $\mathbb{Z}$ is the set of **integer** numbers.
- $\mathbb{N}$ is the set of **natural number.** This is the set of positive integers {1, 2, 3, 4, …}.
- ξ is the **universal set**. The universal set is the whole set we are considering in a question or problem. It is used frequently in Venn diagrams.
- A ⊂ B means that A is a **subset** of B. This means every element in set A must also be an element in set B.

Set notation can be used to describe a situation. For example,

$$Z = \{x : x \text{ } is \text{ } a \text{ } factor \text{ } of \text{ } 20\}$$

means Z is the set of numbers x such that x is a factor of 20, i.e. Z is the set of factors of 20. It could also be written as $Z = \{1, 2, 4, 5, 10, 20\}$.

Tables and Grids

Types of tables and grids can be used to show the outcomes of **combined events**. This is when two events have outcomes that are considered **together**, instead of separately.

Event X has outcomes A, B and C.

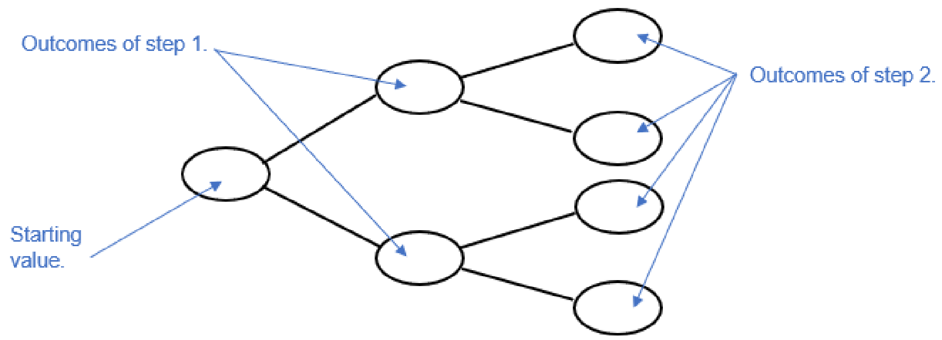
Event Y has outcomes 1, 2 and 3.

		Event X		
		A	B	C
Event Y	1	A1	B1	C1
	2	A2	B2	C2
	3	A3	B3	C3

When Event X has outcome B and Event Y has outcome 2, the combined outcome is B2.

Tree Diagrams

Frequency trees are a visual method of recording the possible outcomes of an event with **multiple steps**. You can use the tree to find the probability of any outcome occurring.



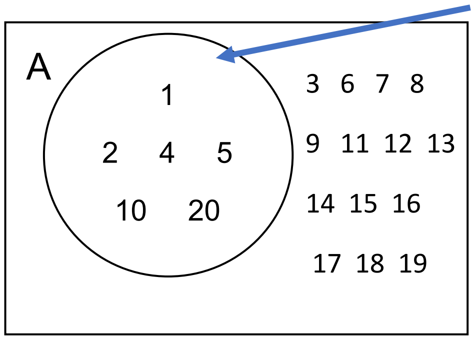
Venn Diagrams

A Venn diagram is a visual method of showing the relationship between sets. It uses circles to group elements of sets together, which often overlap to show shared sub-groups.

A Venn diagram begins with the **universal set**, shown by a rectangular box and labelled with the symbol ξ. This means that the box contains every element of the given set. Within the universal set there are subsets, shown inside circles.

ξ

This universal set symbol denotes all numbers 20 and below. This could also be written $\{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12,\ 13,\ 14,\ 15,\ 16,\ 17,\ 18,\ 19,\ 20\}$.



The subset A includes all the factors of 20. It could also be written

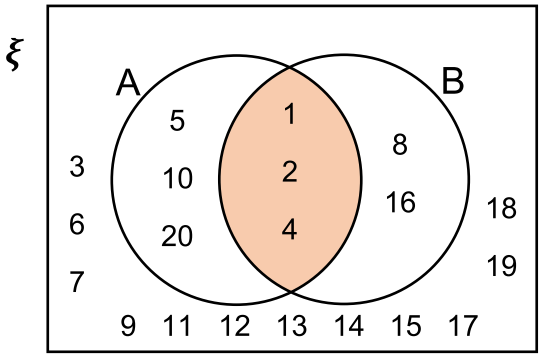
$A = \{1, 2, 4, 5, 10, 20\}$

where $A \subset \xi$.

Intersect (∩)

If a set has multiple sub-sets, their circles can overlap in a Venn diagram. This is the **intersect**, and contains elements that are in set A **and** set B – which is written as **A ∩ B**. In the diagram below, **A ∩ B** is shaded.

- ξ is the set of numbers less than 20: $\xi = \{1, 2, 3, \dots 18, 19, 20\}$
- A is the set of factors of 20: $A = \{1, 2, 4, 5, 10, 20\}$
- B is the set of factors of 16: $B = \{1, 2, 4, 8, 16\}$

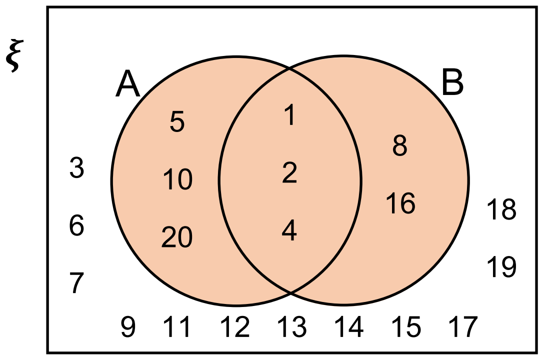


The Venn diagram would look like this, with the common terms of A and B in the intersection. ξ contains everything within the box, and the circles represent subsets A and B.

The set $\boldsymbol{A} \cap \boldsymbol{B} = \{1, 2, 4\}$.

Union (∪)

The **union** on a Venn diagram contains the numbers that are in set A, **or** set B, **or both**. This is written as **A ∪ B**. In the diagram below, **A ∪ B** is shaded. It contains everything within the two circles, as well as the overlap.

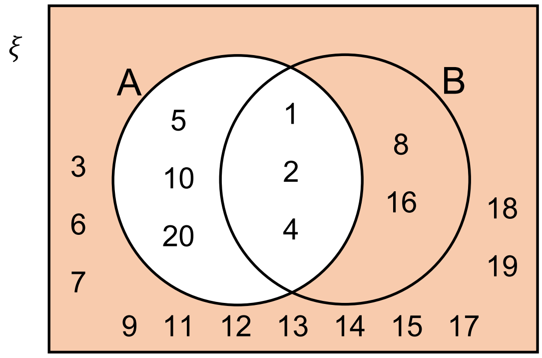


ξ contains everything within the box, and the circles represent subsets A and B.

The set $\boldsymbol{A} \cup \boldsymbol{B} = \{1, 2, 4, 5, 8, 10, 16, 20\}$.

Complement/Not (A′)

The **complement** on a Venn diagram contains the numbers that are **not** in set A – this is written as **A′**. In the diagram below, **A′** is shaded. It contains any part of the diagram that is **not** included in set A's circle.



ξ contains everything within the box, and the circles represent subsets A and B.

The set $\boldsymbol{A'} = \{3, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$.

Three-Way Venn Diagrams

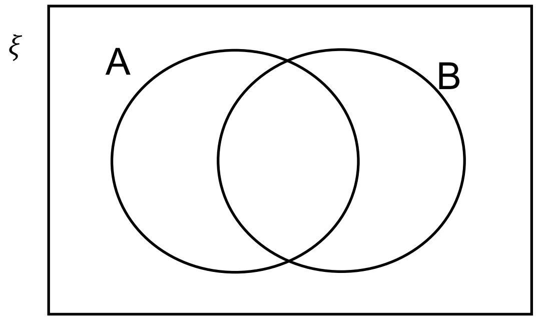
Venn diagrams can also represent **three sets** if they have three circles. The same rules about intersect, union and complement apply:

- **A** ∩ **B** ∩ **C** is the overlap between all three circles.
- **A** ∪ **B** ∪ **C** is anything inside any circle.
- **A** ∩ **C** is the overlap between just A and C circles.
- **A′** is anything in the diagram that is not in set A circle.

Combinations of Sub-Sets

In some questions, sub-sets may be **combined** to create more complicated rules. You may be asked to **shade the region** that represents a particular sub-set. You should work out which areas need to be shaded by **breaking down** the instructions for the sub-set and shading the areas that you know. Combine them to answer the question.

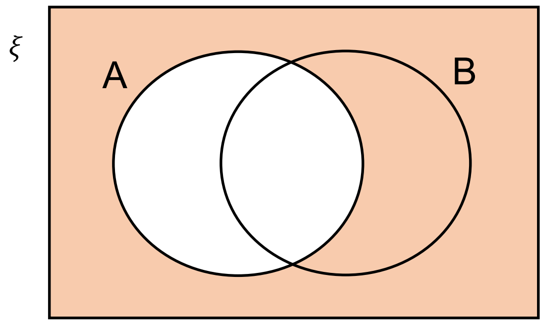
Example: Shade the area on the Venn diagram that represents $A' \cup B$.



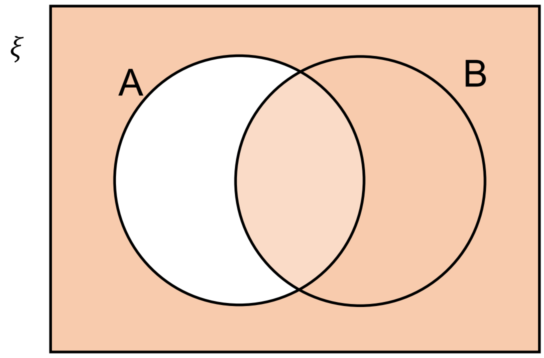
Break the instruction down into steps.

Part 1: Shade region A'
Part 2: Shade region B

Shade the first part on the diagram: Shade the region A'

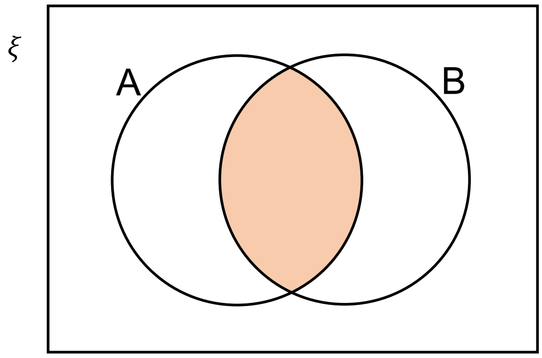


Shade the second part on the diagram: Shade the region B

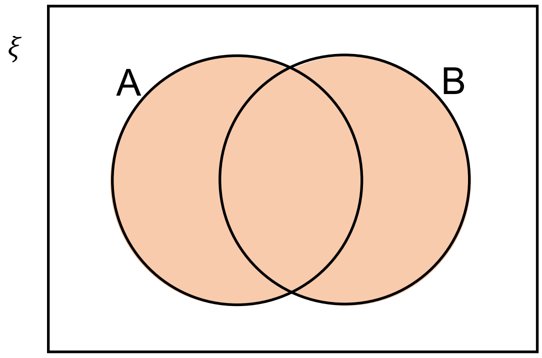


This area includes both parts of the instruction, so represents $A' \cup B$.

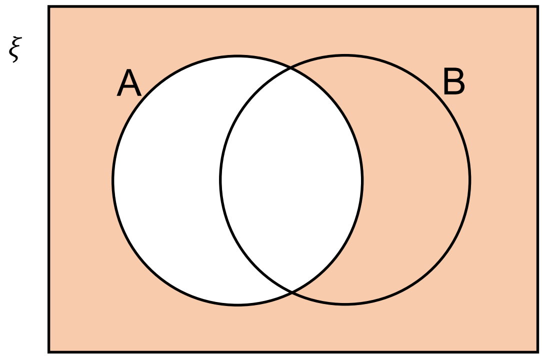
Summary of Venn Diagram Areas



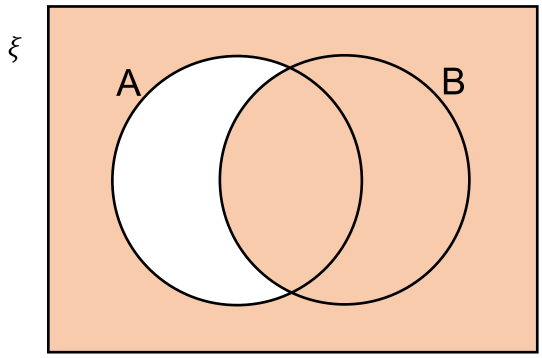
A ∩ B = intersection of A and B



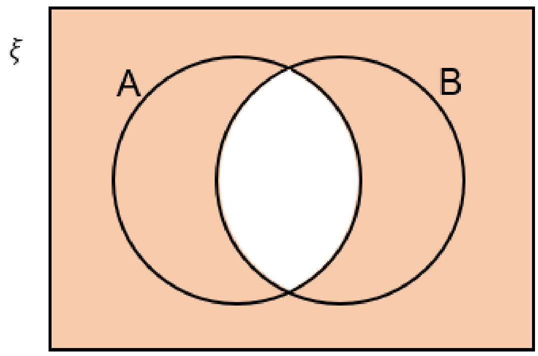
A ∪ B to mean union of A and B



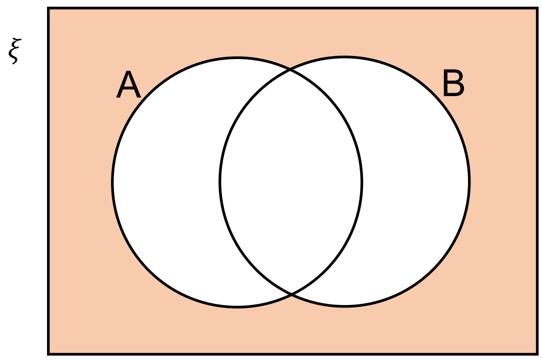
A′ = everything not in A



A′ ∪ B = the union of A′ and B



(A ∩ B)′ = everything not in the intersection

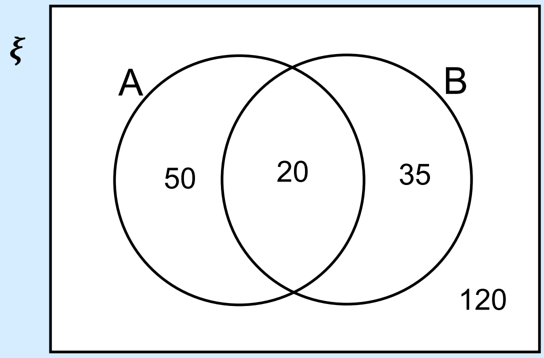


(A ∪ B) ′ = everything not in the union

Example: In the Venn diagram,

$$\xi = \text{people in a village}$$
$$A = \text{women}$$
$$B = \text{football players}$$

a) How many people live in the village?
b) Write down P(A ∩ B).
c) A person from the town is chosen at random. What is the probability that they don't play football?



a) Add up the number of people in the Venn diagram. Don't count the overlap twice, as it applies to set A and B.

$$50 + 20 + 35 + 120 = 225$$

There are $\mathbf{225}$ *people in the village.*

b) Find the number of people in the intersect $\mathrm{A} \cap \mathrm{B}$. Use this value to calculate the probability $\mathrm{P}(\mathrm{A} \cap \mathrm{B})$.

The intersect is the area of overlap. The number in the overlap is 20, $A \cap B = 20$*. There are 225 people who live in the village, so the probability is*

$$P(A \cap B) = \frac{20}{225} = \frac{\mathbf{4}}{\mathbf{45}}$$

c) *Find the number of people who don't play football. This is everyone not in set B. Use this value to calculate the required probability of selecting someone who doesn't play football.*

$$B': 50 + 120 = 170$$

$$P(don't\ play\ football) = \frac{170}{225} = \frac{\mathbf{34}}{\mathbf{45}}$$

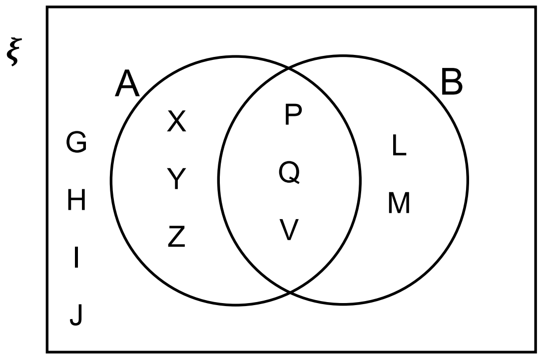
Enumeration, Venn Diagrams, Tree Diagrams and Tables – Practice Questions

1. Janae has 4 shirts, 2 pairs of jeans and 2 pairs of shoes. How many outfits can she make?

2. Maia flips a coin and rolls a 6-sided die. Fill in the table to show all the possible outcomes.

		Die					
		1	2	3	4	5	6
Coin	Heads						
	Tails						

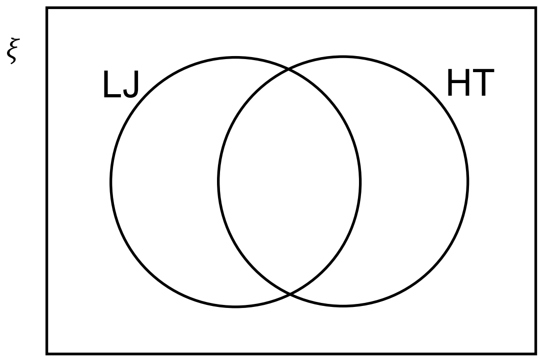
3. Here is a Venn diagram. Write down the letters that are in $A \cap B$:



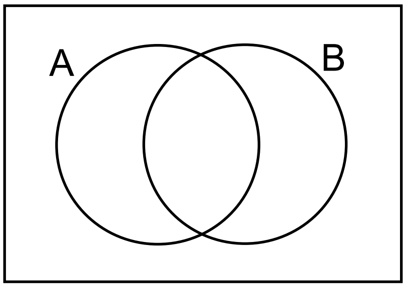
4. There are 60 members in an athletics club.

   9 athletes do long jump and hammer throw.
   20 athletes do only long jump.
   14 athletes do not do long jump or hammer throw.

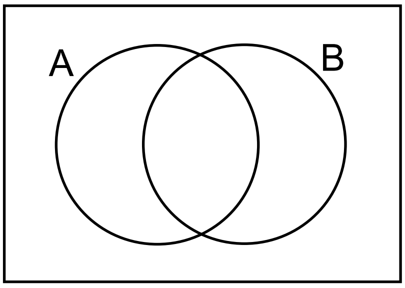
   Complete the Venn diagram, working out how many athletes do only hammer throw.



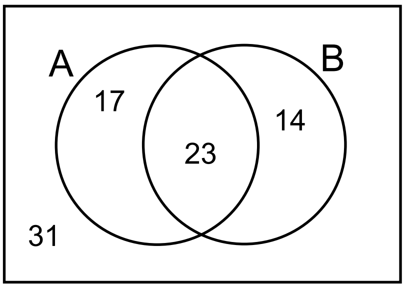
5. Shade the area B':



6. Shade the area A' ∩ B:



7. From the Venn diagram, find P(A' ∪ B).



8. Juliana goes to the local leisure centre. She records that 50 people visit that day. Of those that visit,

30 visited the gym
13 people went swimming
22 people went to the spa
6 people went swimming and went to the spa
11 people went to the spa and visited the gym
12 people went swimming and visited the gym
5 people went swimming, visited the gym and went to the spa

a) Draw a Venn diagram to display this information.

b) How many people visited the leisure centre but did not visit any of the gym, spar or swimming pool?

c) One person is picked out. What is the probability that they visited the gym and went swimming?

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.